



ACL-1264

Seat No. _____

B. Sc. (Sem. I) Examination

November / December – 2016

Mathematics : Paper-CC-MAT-111

Time : 3 Hours]

[Marks : 70

Instructions: (1) All Questions are Compulsory.

(2) The figures to the right indicate marks of the corresponding question.

1 (a) State and Prove: Leibnitz's Theorem

[7]

OR

(a) Expand the function $f(x) = \log(1+x)$, $-1 < x \leq 1$ in ascending powers of x .

[7]

(b) Attempt any two.

[8]

(1) If $I_n = \frac{d^n}{dx^n} (x^n \log(x))$ then prove that $I_n = nI_{n-1} + (n-1)!$ and from it deduce that

$$I_n = n! \left[\log x + 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n} \right].$$

(2) Using Cauchy mean value theorem prove that $b^b \cdot a^a = c^c$ ($\log b - a \log a$), ($0 < a < c < b$)

(3) Expand $\tan^{-1} x$ in ascending powers of x .

2(a) Obtain formula for length of arc of continuous curve $y=f(x)$ between lines $x=a$ and $x=b$.

[7]

OR

(a) Obtain reduction formula for $\int \sin^m x \cos^n x dx$, $n \in \mathbb{N}$ and hence deduce reduction

formula for $\int_0^{\pi/2} \sin^m x \cos^n x dx$.

[7]

(b) Attempt any two.

[8]

(1) Evaluate: $\int_0^1 x^4 (2-x^2)^{3/2} dx$

(2) Evaluate: $\lim_{n \rightarrow \infty} \left[\left(1 + \frac{1}{n^2}\right) \left(1 + \frac{2}{n^2}\right)^2 \left(1 + \frac{3}{n^2}\right)^3 \dots \left(1 + \frac{n}{n^2}\right)^n \right]^{1/n^2}$

(3) Find the volume of sphere of radius a .

3(a) For vectors \vec{a}, \vec{b} and \vec{c} prove that (1) $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$

$$(2) (\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{b} \cdot \vec{c})\vec{a} \quad [7]$$

OR

(a) Prove that: $\text{Curl}(\Phi \vec{F}) = (\text{grad} \Phi) \times \vec{F} + \Phi(\text{curl} \vec{F})$, where Φ is a scalar and \vec{F} is a vector function.

[7]

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[Contd...

(b) Attempt any two.

[8]

(1) Find reciprocal vector set for the vector set $\{(1,1,-1), (1,-1,1), (-1,1,1)\}$

(2) Find divergence and curl of a vector function $\vec{F} = xyz\vec{i} + 3x^2y\vec{j} + (xz^2 - y^2z)\vec{k}$ at point $(2,-1,1)$.

(3) Transform the equation $x^2 + y^2 = z^2$ into cylindrical and spherical co-ordinates.

4 (a) Derive the equation of a tangent plane of sphere $x^2 + y^2 + z^2 = a^2$ at point (α, β, γ) . [7]

OR

(a) Equation of right circular cone having vertex (α, β, γ) , axis $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ and semi vertical angle θ in R^3 . ($\theta \neq 0, \theta \neq \pi/2$) [7]

(b) Attempt any two.

[8]

(1) Find the value of k , if plane $kx + y - 2z = 9$ touches the sphere $x^2 + y^2 + z^2 = 9$.

(2) Define Orthogonal Sphere. Prove that two sphere $x^2 + y^2 + z^2 - 2x + 4y - 4 = 0$ and $x^2 + y^2 + z^2 - 6y + 4z = 0$ are orthogonal.

(3) Find the equation of right circular cylinder having guiding curve $x+y+z=-3$, $x^2 + y^2 + z^2 + 3x + 3y + 3z = 0$.

[10]

5 Attempt any five.

(1) If $y = \frac{x}{x^2 - 2x - 8}$, $x \neq 2, -4$ then find y_n .

(2) Find the coefficient of x^4 in the expansion of $\log(\cos x)$.

(3) Evaluate: $\int_0^\pi \sin^3\left(\frac{\theta}{2}\right) d\theta$.

(4) Find the limit of series $\frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \dots + \frac{1}{n+n} + \dots$

(5) For vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ prove that

$$(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) + (\vec{b} \times \vec{c}) \cdot (\vec{a} \times \vec{d}) + (\vec{c} \times \vec{a}) \cdot (\vec{b} \times \vec{d}) = 0.$$

(6) Prove that $\nabla r^2 = 2\vec{r}$, where $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $r = |\vec{r}|$.

(7) Find the equation of sphere whose extremities of diameter are $(3,4,0)$ and $(2,3,-1)$.

(8) Define: Central Conicoids, Ellipsoid.

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[6930]