MDD-4264 Seat No

B. Sc. (Sem. I) Examination

November / December - 2018

Mathematics : CC - MATH - 111

Time : 3 Hours] [Total Marks 70

Instructions: (1) All questions are compulsory.

(2) Figure to the right indicates the marks of the corresponding question.

1 (a) State and prove Liebnitz's Theorem. 06

OR

(a) State and prove Cauchy's theorem. 06

(b) Attempt any two. 10

1. If $y = e^{ax}\cos(bx+c)$, $a,b,c \in R$ then prove that

$y_n = r^n e^{ax}\cos(bx+c-n\alpha)$,

where $a = r\cos\alpha$, $b = r\sin\alpha$, $r = \sqrt{a^2+b^2}$, $\alpha = \tan^{-1}\frac{b}{a}$.

2. Prove that $\log(1+x^2) = -x^2 - \frac{x^4}{2} - \frac{x^6}{3} - \ldots$

3. If $y = \cos^{-1}x, x \in (-1,1)$ then prove that

$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$.

(a) For $n \in N$, Obtain reduction formula 07

$$\int \cos^n x\,dx = -\frac{1}{n}\cos^{n-1}x\sin x + \frac{n-1}{n}\int \cos^{n-2}x\,dx.$$

From this prove that if $I_n = \int_0^{\frac{\pi}{2}} \cos^n x\,dx$ then $I_n = \frac{n-1}{n} I_{n-2}$

OR

(a) Prove that formula of length of arc is $S = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$.

(b) Attempt any two.

1. $\lim\left[\frac{1}{1^2+n^2} + \frac{2}{2^2+n^2} + \frac{3}{3^2+n^2} + \ldots\ldots + \frac{n}{n^2+n^2}\right]$.

2. Evaluate $\int_0^1 x^4(2-x^2)^{\frac{3}{2}}\,dx$.

3. Find the length of cycloid equation is
$x = a(\theta - \sin\theta), y = a(1-\cos\theta)$.

3 (a) Vectors $\bar{a},\bar{b},\bar{c}$ are coplanar if and only if $\left[\bar{a}\ \ \bar{b}\ \ \bar{c}\right] = 0$.

OR

(a) Obtain the polar equation of a straight line passing through the points (r_1,θ_1) and (r_2,θ_2).

(b) Attempt any two.

1. Prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r}f'(r)$, where $r = |\bar{r}|$.

2. Find the centre and radius of circle
$x^2+y^2+z^2-2y+2z-23=0, x+2y-2z+5=0$

3. Find out reciprocal vector set of the set $\{(4,1,2),(2,-1,1),(-1,-1,1)\}$

4 (a) If the plane $lx+my+nz=p$ touches the sphere
$x^2+y^2-z^2=a^2$
obtain the condition and point of contact.

OR

(a) Obtain equation of the tangent plane to a sphere
$x^2+y^2+z^2=a^2$ at point $P(\alpha,\beta,\gamma)$.

(b) Attempt any two.

1. Find the value of k if the plane $2x-y-3z=k$ touches to the sphere $x^2+y^2+z^2=7$

2. Find the equation of circle through the point (0,0,0),(0,0,1), (2,0,0) and (0,3,0).

3. Find the equation of the sphere passing through the circle $x^2 + y^2 + z^2 = 9, 2x + 3y + 4z = 5$ and point (1,2,3).

5 Attempt any two.

1. The functions $f(x) = x^3$, & $g(x) = x^2$, $x > 1$ then prove that $\frac{3}{2} < \frac{x^3 - 1}{x^2 - 1} < \frac{3}{2}x$. **08**

2. If $F(x, y, z) = x^2\,\bar{i} + y^2\,\bar{j} + z^2\,\bar{k}$ then prove that $curl\ F = 0$.

3. Find the equation of a cylinder whose axis is $\frac{x}{1} = \frac{y}{-1} = \frac{z}{-1}$ and the guiding curve is $2x^2 + 3y^2 = 1, z = 0$.