



ABC-2967

Seat No. \_\_\_\_\_

**B. Sc. (Sem. II) Examination**

March / April - 2019

**Mathematics : CCMATH-122**

Time : 3 Hours]

[Total Marks : 70

- (a) State and prove De-Moiver's theorem. 7

**OR**

Prove that

$$\cos n\theta = \cos^n \theta - \binom{n}{2} \cos^{n-2} \theta \cdot \sin^2 \theta + \binom{n}{4} \cos^{n-4} \theta \cdot \sin^4 \theta - \dots$$

where  $n \in N$ .

- (b) Attempt any two : 8

- (1) If  $x^2 - 2x \cos \theta + 1 = 0$  then prove that,  
 $x^{2n} - 2x^n \cos n\theta + 1 = 0$ .

- (2) Prove that :

$$\sin^7 \theta = \frac{1}{64} [35 \sin \theta - 21 \sin 3\theta + 7 \sin 5\theta - \sin 7\theta].$$

- (3) Construct the equation whose roots are

$$2 \cos \frac{\pi}{7}, 2 \cos \frac{3\pi}{7} \text{ and } 2 \cos \frac{5\pi}{7}.$$

- 2 (a) Prove that :  $\cosh^{-1}(z) = \log(z + \sqrt{z^2 - 1})$ .

**OR**

Show that :  $\sum \frac{n^p}{\sqrt{n+1} - \sqrt{n}}$  is convergent if

$p < -\frac{1}{2}$  and it is divergent if  $p \geq -\frac{1}{2}$ .

- (b) Attempt any two :

- (1) If  $\cosh(x) = \sec \theta$  then prove that

$$\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right) = e^x.$$

- (2) If  $i^{\alpha-i\beta} = \alpha + i\beta$  then prove that

$$\alpha^2 + \beta^2 = e^{-(4n+1)\beta\pi}.$$

- (3) Discuss the convergence of the following series :

$$\sum \left(1 - \frac{1}{n}\right)^{n^2} \text{ and } \sum \left(\sqrt[n]{n} - \frac{1}{2}\right)^n x^n.$$

- 3 (a) Explain the general solution of Bernoulli's differential equation.

**OR**

Prove that  $\frac{1}{f(D)}(e^{ax} \cdot V) = e^{ax} \cdot \frac{1}{f(D+a)} V$ .

$V$  is a function of  $x$ .

(b) Solve any two :

(1)  $y\sqrt{1+p^2} = (x+yp)^2$

(2)  $D^3(D^2-1)y = 5x^2$

(3)  $y'' - 5y' + 6y = x^2e^x$

(a) If  $A = [a_{ij}]_{n \times n}$  then prove that,

$A(adj A) = (adj A)A = |A|I_n$

OR

If A and B are matrices of order  $m \times n$  and  $n \times p$  respectively then, prove that  $(AB)^T = B^T A^T$ .

(b) Attempt any two :

(1) Find inverse of a matrix  $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$  by

Row-Reduction method.

(2) Obtain the rank of matrix

$B = \begin{bmatrix} 3 & 0 & 0 & 2 \\ 1 & 2 & -4 & -1 \\ 0 & -3 & 6 & 1 \end{bmatrix}$

(3) Give an example of a Hermitian and a Skew-Hermitian matrix of order 3.

5 Attempt any five :

(1) Find  $|z|$  and principal Arg of  $z = (1+i)^{1-i}$

(2) Solve by De-Moiver's theorem :

$x^3 - x^2 + x - 1 = 0$

(3) Solve :  $\tanh z + 2 = 0$

(4) Write the P.I. of differen.

$(D^3 + D^2 - D - 1)y = \cos 2x$

(5) Obtain the radius of convergence of

$\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^{n^2} \cdot x^n$

(6) Solve :  $p^3 - 4xyp + 8y^2 = 0$

(7) Transform the matrix  $\begin{bmatrix} 2 & -1 & 4 \\ 3 & 0 & -1 \\ 1 & 0 & 3 \end{bmatrix}$  into the

Echelon form by row operations.

(8) If A is a square matrix then prove that  $A + A^*$  is Hermitian and  $A - A^*$  is Skew-Hermitian matrix.