



CCU-622      Seat No. \_\_\_\_\_  
**B. Sc. (Sem. II) Examination**  
 October - 2019  
**CC MATH - 122 : Mathematics**

Time : 2:30 Hours ]

[ Total Marks : 70

**Instructions :** (1) All questions are compulsory.  
 (2) Figures to the right indicates the marks of the corresponding question.

1 (a) If  $n$  is any rational number then 8

$$(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta \text{ where } i^2 = -1.$$

OR

(a) If  $q$  be any positive integer then show that 8

$$(\cos \theta + i \sin \theta)^{1/q} \text{ has only } q \text{ different values.}$$

(b) Attempt any two : 10

$$(1) \text{ Prove that } \left( \frac{1 + \sin \theta + i \cos \theta}{1 + \sin \theta - i \cos \theta} \right)^n =$$

$$\cos\left(\frac{n\pi}{2} - n\theta\right) + i \sin\left(\frac{n\pi}{2} - n\theta\right)$$

(2) If  $\alpha, \beta$  are the roots of  $x^2 - 2x + 4 = 0$

$$\text{then prove that } \alpha^n + \beta^n = 2^{n+1} \cos \frac{n\pi}{3}.$$

(3) Expand  $\sin n\theta, \cos n\theta$  in the power of  $\sin \theta, \cos \theta$  respectively where  $n \in N$ .

2 (a) Series  $\sum \frac{1}{n^p}$  is convergent if  $p > 1$  and is divergent if  $p \leq 1$ . 7

OR

(a) Suppose  $\Sigma a_n$  is an infinite series of positive numbers and  $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = l$ . 7

- (1) If  $0 < l < 1$ , then  $\Sigma a_n$  is convergent.
- (2) If  $l > 1$ , then  $\Sigma a_n$  is divergent.
- (3) If  $l = 1$ , then test is inconclusive.

(b) Answer any two : 10

- (1) If  $i^{n+i\beta} = \alpha + i\beta$  then prove that  $\alpha^2 + \beta^2 = e^{-(4n+1)\beta}$ .
- (2) Discuss the convergent of series

$$\sum \frac{x^n}{(n+1)(n+2)}.$$

(3) Find the radius of convergence

$$\sum_{n=1}^{\infty} \frac{1+2+3+\dots+n}{n!} x^n.$$

3 (a) Discuss the solution of linear differential equation  $\frac{dy}{dx} + Py = Q$ ,  $P$  and  $Q$  are function of  $x$  8

OR

(a) Prove in usual notation

$$\frac{1}{f(D)} e^{ax} F(x) = e^{ax} \frac{1}{f(D+a)} F(x)$$

(b) Solve any two differential equations : 10

$$(1) (1+x^2) \frac{dy}{dx} + y = \tan^{-1} x$$

$$(2) (D^2 - 4D + 4)y = \sin 3x$$

$$(3) (D^2 - 5D + 6)y = x^2 e^x$$

4 (a) If  $A = [a_{ij}]_n$  and  $B = [b_{ij}]_n$  are square 7

matriees then prove that -

(1) If  $A$  and  $B$  are symmetric then  $A + B$  is symmetric.

(2) If  $A$  and  $B$  are Skew symmetric then  $A + B$  is Skew symmetric.

OR

(a) Define Transpose of matrix. If  $A$  and  $B$  are  $m \times n$  and  $n \times p$  matrices respectively then prove that  $(AB)^T = B^T A^T$ . 7

(b) Attempt any two : 10

$$(1) \text{ If } A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} \text{ then (i) find } A^{-1}$$

$$\text{(ii) show that } A^{-1} = A^3$$

(2) Solve the system of equations :

$$x + 3y + 6z = 2$$

$$3x - y + 4z = 9$$

$$x - 4y + 2z = 7$$

(3) Find the rank of matrix

$$A = \begin{bmatrix} 3 & 2 & 0 & -1 \\ 1 & -1 & 2 & 2 \\ 0 & 1 & -3 & -1 \end{bmatrix}$$

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