



KC-487

Seat No. _____

B. Sc. (Sem. V) Examination

October / November - 2017

Mathematics : CC-MATH-503

(Differential Equations)

Time : 3 Hours]

[Total Marks : 70

- Instructions :
- (1) All questions are compulsory.
 - (2) Figures to the right indicate the marks of corresponding question.

1 (a) Prove that 6

$$\frac{1}{f(D)} [e^{ax} \cdot v] = e^{ax} \cdot \frac{1}{f(D+a)} v$$

where $a = \text{constant}$, $v = \text{function of } x$.

(b) Solve : $(D^3 - 5D^2 + 7D - 3)y = e^{2x} \cdot \cosh x$. 6

(c) Solve : $(D^4 + D^3 + D^2)y = x^2 (a + bx)$. 6

OR

1 (a) Prove that 6

(1) $\frac{1}{D^2 + a^2} \cos ax = \frac{x}{2a} \sin ax$

(2) $\frac{1}{D^2 + a^2} \sin ax = \frac{-x}{2a} \cos ax$

(b) Solve : $(D^2 + 2)y = x^2 \cdot e^{3x}$. 6

(c) Solve : $(D^2 - 1)y = x^2 \cdot \cos x$. 6

2 (a) If the Linear differentiable equation 6

$$P_0 \cdot y^{(n)} + P_1 y^{(n-1)} + P_2 y^{(n-2)} + \dots + P_n \cdot y = \phi(x),$$

where $P_0, P_1, P_2, \dots, P_n$ are function of x , is exact differential equation then prove that

$$P_n + (-1) P_{n-1}^{(1)} + (-1)^2 \cdot P_{n-2}^{(2)} + \dots + (-1)^n P_0^{(n)} = 0.$$

(b) Solve : $x^2 y_2 + (xy_1 - y)^2 - 3y^2 = 0$ 6

where $y_1 = \frac{dy}{dx}$, $y_2 = \frac{d^2y}{dx^2}$.

(c) Solve : 6

$$yy^{(2)} = \{y^{(1)}\}^2 [1 - y^{(1)} \cos y + yy^{(1)} \sin y].$$

OR

2 (a) Show that the equation 6

$$x^2 \cdot y^{(3)} + 4x \cdot y^{(2)} + (x^2 + 2)y^{(1)} + 3xy = 2$$

becomes exact on being multiplied by some power of x . Obtain its first integral.

(b) Solve : $y^{(2)} = y^3 - y$ given that $y^{(1)} = 0$ for $y = 1$. 6

(c) Solve : 6

$$\left\{ y^{(1)} \right\}^2 - y^{(2)} = n \left[\left\{ y^{(1)} \right\}^2 + a^2 \left\{ y^{(2)} \right\}^2 \right]^{\frac{1}{2}}$$

3 (a) Solve : $xy^{(2)} - (x-2)y^{(1)} - 2y = x^3$. 6

(b) Solve : 6

$$y^{(2)} - 4x \cdot y^{(1)} + (4x^2 - 1)y^{(1)} + y = -3e^{x^2} \cdot \sin x$$

(By Normal Form)

(c) Solve : 6

$$(1+x)^2 \cdot y^{(2)} + (1+x)y^{(1)} + y = 4 \cos(\log(1+x))$$

[By changing the independent variable]

OR

3 (a) Solve : $xy^{(2)} - (x-2)y^{(1)} - 2y = x^3$. 6

(b) Solve : $xy^{(2)} - (1+x)y^{(1)} + y = x^2$. 6

[By factorisation of the operator]

(c) Solve : $y^{(2)} + a^2 y = \sec ax$ 6

[By variation of parameter]

4 Solve any four :

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(1) $(D^3 - 2D^2 - 4D + 8)y = 0$

(2) $(D^2 - 5D + 6)y = \sin 3x$

(3) $y^{(3)} = x - \sin x$

(4) $y^{(3)} + x \cdot y^{(3)} - y^{(2)} = 0$

(5) $y^{(2)} + 2y^{(1)} + 2y = 1 + x^2$

[By method of undetermined coefficients]

(6) $(D^2 - 9D + 18)y = e^{-3x}$