CAF-131

Seat No. ______

B. Sc. (Sem.- V) Examination

December – 2021

CC MATH-502 : Mathematics

(Mathematical Analysis - I)

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

Instructions : (1) All questions are compulsory.
(2) Figure to the right indicates full mark of the corresponding question.

1 (a) Prove that there is no rational number whose square is 12 **6**

(b) Prove that R has the least-upper-bound property. **6**

(c) State and Prove : Archimedean property of $\mathbb{R}$. Also, show that between any two real numbers there is a rational one. **6**

OR

(a) Suppose S is an ordered set with least-upper-bound-property. B is non empty subset of S such that B is bounded below. Let L be the set of all lower bounds of B. Then Prove that sup L is exist in S and sup L= Inf B. **6**

(b) Show that for every real x>0 and every integer n>0, there is one and only one positive real y such that $y^n = x$. **6**

(c) State and Prove Schwarz inequality for complex numbers z_i and w_i; where i=1, 2, 3...., n. **6**

2 (a) Define countable and uncountable set **6**

Let A be the set of all sequences whose elements are digits 0 and 1

i.e. Elements of A are sequences like 1, 0, 0, 1, 0, 1, 1, 1, ...

Then show that A is uncountable Set

(b) If X is a metric space and E is subset of X then prove that **6**

(i) $\overline{E}$ is closed

(ii) $E = \overline{E}$ if and only if E is closed

(iii) $\overline{E} \subset F$ for every closed set $F \subset X$ such that $E \subset F$

(c) If a set E is $\mathbb{R}^k$. Prove that following statements are equivalent **6**

(i) E is closed and bounded

(ii) E is compact

(iii) Every infinite subset of E has limit paint in E

OR

(a) Let (X,d) the metric space and $Y \subset X$. Then Prove that A subset E of Y is open relative to Y if and only if $E = Y \cap G$ for some open subset G of X **6**

(b) Prove that every k-cell is compact. **6**

(c) Define connected set **6**

$E \subset \mathbb{R}$ is connected if and only if it has property : If $x \in E$, $y \in E$ and $x<z<y$, then $z \in E$

3 (a) Suppose $\{a_n\}, \{b_n\}$ are sequences of complex numbers such that $\lim_{n \to \infty} a_n = a$ and $\lim_{n \to \infty} b_n = b$ then prove that **6**

(i) $\lim_{n \to \infty} a_n b_n = ab$

(ii) $\lim_{n \to \infty} \frac{1}{a_n} = \frac{1}{a}$, Where

$a_n \neq 0 \ (n = 1, 2, 3, \ldots), a \neq 0$

(b) Define subsequence with example. **6**

Prove that the subsequential limits of a sequence $\{p_n\}$ in a metric space X from a closed subset of X.

(c) Let (X,d) be metric space and X is compact then prove that "Every cauchy sequence is convergent" and conversely. **6**

OR

(a) State and prove root test for series. **6**

(b) Prove that $\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = e$ **6**

(c) Suppose that $a_1 \geq a_2 \geq a_3 \geq \ldots \geq 0$. Then **6**

Prove that $\sum_{n=1}^{\infty} a_n$ converges if and only if $\sum_{k=0}^{\infty} 2^k a_{2^k}$ converges.

4 Attempt any two questions. **8**

(i) State and prove parallelograms for $x, y \in R^k$ **4**

(ii) Define Cantor set and show that it is perfect set. **4**

(iii) Let (X, d) be a metric space and $E \subset X$. Then prove that E is open $\Leftrightarrow$ Complement of E is closed. Also check the set of all prime numbers in R is open or closed ? **4**

5 Attempt any **two** questions. **8**

(i) Define $\lim_{n \to \infty} \sup a_n$ and $\lim_{n \to \infty} \inf a_n$ and find for following sequences. **4**

(a) a_n is sequence containing all rationals.

(b) $a_n = \frac{(-1)^n}{\left(1 + \frac{1}{n}\right)}$

(ii) Define e and show that that e is an irrational number. **4**

(iii) Find the radius of convergence of following series. **4**

(a) $\sum \frac{2^n}{n!} x^n$

(b) $\sum \frac{n^3}{3^n} x^n$
